

Curvaton reheating mechanism in a Scale Invariant Two Measures Theory

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Abstract

The curvaton reheating mechanism in a Scale Invariant Two Measures Theory defined in terms of two independent non-Riemannian volume forms (alternative generally covariant integration measure densities) on the space-time manifold which are metric independent is studied. The model involves two scalar matter fields, a dilaton, that transforms under scale transformations and it will be used also as the inflaton of the model and another scalar, which does not transform under scale transformations and which will play the role of a curvaton field. Potentials of appropriate form so that the pertinent action is invariant under global Weyl-scale symmetry are introduced. Scale invariance is spontaneously broken upon integration of the equations of motion. After performing transition to the physical Einstein frame we obtain: (i) For given value of the curvaton field an effective potential for the scalar field with two flat regions for the dilaton which allows for a unified description of both early universe inflation as well as of present dark energy epoch; (ii) In the phase corresponding to the early universe, the curvaton has a constant mass and can oscillate decoupled from the dilaton and that can be responsible for both reheating and perturbations in the theory. In this framework, we obtain some interesting constraints on different parameters that appear in our model; (iii) For a definite parameter range the model possesses a non-singular “emergent universe” solution which describes an initial phase of evolution that precedes the inflationary phase. Finally we discuss generalizations of the model, through the effect of higher curvature terms, where inflaton and curvaton can have coupled oscillations.

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I. INTRODUCTION

Inflationary universe models [1] have solved some problems of the Standard Hot Big Bang scenario, like the horizon and flatness problems. High among the important accomplishments of inflation are a natural source of the primordial perturbations [2]. Inflation represents a period of exponential accelerated expansion and it so happens that the present Universe is also undergoing a period of accelerated expansion [3] ,[4], although much more slowly. The possibility of continuously connecting an inflationary phase to a slowly accelerating universe through the evolution of a single scalar field – the *quintessential inflation scenario* – has been first studied in Ref.[5]. Also, $F(R)$ models can yield both an early time inflationary epoch and a late time de Sitter phase with vastly different values of effective vacuum energies [6]. For a recent proposal of a quintessential inflation mechanism based on the k-essence [7] framework, see Ref.[8]. For another recent approach to quintessential inflation based on the “variable gravity” model [9] and for extensive list of references to earlier work on the topic, see Ref.[10].

In a sequence of previous papers [11], [12], [13] we approached the question of continuously connecting an inflationary phase to a slowly accelerating universe through the evolution of a single scalar field in the context of the Two Measures Theories [14]- [20]. Two Measures Theories models which use a non Riemannian measure of integration in the action, and in the case of a scale invariant theory, the scale invariance was spontaneously broken by the equations of motion associated with the degrees of freedom which defined the non Riemannian measure of integration in the action. These degrees of freedom that define a non Riemannian measure of integration could be for example four scalar fields in four dimensions. Models where four scalar fields in four dimensions have been used in the measure of integration and also in other parts of the action were studied by Struckmeier [21]. We also insisted in [11], [12], [13] (also see [24] for a Two Measures Theory that allows an emergent universe scenario, although without unification of inflation and dark energy) in solving the initial singularity problem by having an Emergent scenario. Emergent scenario [25] are non singular cosmological type of scenarios, where the universe starts as an Einstein Universe before developing into the inflationary period. In this context, in the context of scale invariant two measures theories, in our most recent paper [11], we used two non Riemannian measure of integration in the action, the equations of motion of each of the two

measures of integration leading to two independent integration constants, these integration constants break scale invariance, and define the strength of the dark Energy density in the present universe, while they play no role in the early Universe [11].

One problem with the scenarios that connect smoothly connect an inflationary phase to a slowly accelerated phase, is that such models are not oscillating and therefore reheating may be problematic. One solution to this is to introduce another field, the "curvaton" [27], [28]. The possible role of curvaton reheating in Non -Oscillatory Inflationary Models has been in particular studied by [29, 30]. We will study the dynamic of the curvaton field through different scenarios, and obtain the constraints upon the free parameters on our model in order to have a feasible curvaton stage during the reheating of the universe[29, 31]. Firstly we will consider that the curvaton coexists with the inflaton field during the inflationary scenario, here the inflaton energy density is the dominant component, and the curvaton energy density should survive to the expansion of the universe, in which the curvaton field has to be effectively massless. The next scenario the curvaton field gets effectively massive during the kinetic epoch. During this epoch, the curvaton should oscillates in the minimum of its effective potential, and its energy density develops as non-relativistic matter. Finally the curvaton field decay into radiation, and then the standard big bang cosmology is recuperated. At this point, we will study two scenarios for the decay of the curvaton field, since the curvaton field could decays before or after it becomes the dominant energy density of the universe. However, the curvaton field introduces an interesting study for the observed large-scale adiabatic density perturbations in the early universe. The hypothesis of the curvaton field suggests that the adiabatic density perturbation proceeds from the curvaton and not from the inflaton field. In this framework, the adiabatic density perturbation is originated only after the inflationary scenario, and then the initial condition are purely isocurvature perturbations. Recently the curvaton field is applied to the different theories[32]. In this form, we will study the curvaton perturbation for both decays before or after it becomes the dominant energy density of the universe.

In this paper we will see that the two measures theory that was discussed in [11] allows a simple generalization with the addition of a curvaton field as we will see in the next section.

The outline of the paper goes as follow: in Sec. II we give a description of two independent non-Riemannian volume-forms, with the dilaton-inflaton and curvaton fields. In Section III the curvaton field is analyzed in the kinetic epoch. The Section IV describes the curvaton

decay after its domination. The Section V explains the decay of the curvaton field before it dominates, and in Section VI includes our conclusions.

II. THE MODEL: TWO INDEPENDENT NON-RIEMANNIAN VOLUME-FORMS, WITH THE DILATON-INFLATON AND CURVATON FIELDS

We follow the general structure of the paper [11], but now we will enrich the field content of the theory with a new field σ which will not transform under scale transformations, so we write,

$$S = \int d^4x \Phi_1(A) \left[R + L^{(1)} \right] + \int d^4x \Phi_2(B) \left[L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}} \right]. \quad (1)$$

Here the following notations are used:

- $\Phi_1(A)$ and $\Phi_2(B)$ are two independent non-Riemannian volume-forms, *i.e.*, generally covariant integration measure densities on the underlying space-time manifold:

$$\Phi_1(A) = \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_\mu A_{\nu\kappa\lambda} \quad , \quad \Phi_2(B) = \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_\mu B_{\nu\kappa\lambda} \quad , \quad (2)$$

defined in terms of field-strengths of two auxiliary 3-index antisymmetric tensor gauge fields[41]. $\Phi_{1,2}$ take over the role of the standard Riemannian integration measure density $\sqrt{-g} \equiv \sqrt{-\det \|g_{\mu\nu}\|}$ in terms of the space-time metric $g_{\mu\nu}$.

- $R = g^{\mu\nu} R_{\mu\nu}(\Gamma)$ and $R_{\mu\nu}(\Gamma)$ are the scalar curvature and the Ricci tensor in the first-order (Palatini) formalism, where the affine connection $\Gamma_{\nu\lambda}^\mu$ is *a priori* independent of the metric $g_{\mu\nu}$. Note that in the second action term we have added a R^2 gravity term (again in the Palatini form). Let us recall that $R + R^2$ gravity within the second order formalism (which was also the first inflationary model) was originally proposed in Ref.[22].
- $L^{(1,2)}$ denote two different Lagrangians of two scalar fields, the dilaton φ , which will play the role of an inflaton and now also the curvaton σ . The action will be taken of

[41] In D space-time dimensions one can always represent a maximal rank antisymmetric gauge field $A_{\mu_1 \dots \mu_{D-1}}$ in terms of D auxiliary scalar fields ϕ^i ($i = 1, \dots, D$) in the form: $A_{\mu_1 \dots \mu_{D-1}} = \frac{1}{D} \varepsilon_{i i_1 \dots i_{D-1}} \phi^i \partial_{\mu_1} \phi^{i_1} \dots \partial_{\mu_{D-1}} \phi^{i_{D-1}}$, so that its (dual) field-strength $\Phi(A) = \frac{1}{D!} \varepsilon_{i_1 \dots i_D} \partial_{\mu_1} \phi^{i_1} \dots \partial_{\mu_D} \phi^{i_D}$.

the form (similar to the choice in Refs.[14]):

$$L^{(1)} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - \frac{\mu^2\sigma^2}{2}\exp\{-\alpha\varphi\} - V(\varphi), V(\varphi) = f_1\exp\{-\alpha\varphi\}, \quad (3)$$

$$L^{(2)} = -\frac{b}{2}e^{-\alpha\varphi}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + U(\varphi), U(\varphi) = f_2\exp\{-2\alpha\varphi\} \quad (4)$$

where α, f_1, f_2, mu^2 are dimensionfull positive parameters, whereas b is a dimensionless one.

- $\Phi(H)$ indicates the dual field strength of a third auxiliary 3-index antisymmetric tensor gauge field:

$$\Phi(H) = \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu H_{\nu\kappa\lambda}, \quad (5)$$

whose presence is crucial for non-triviality of the model.

The scalar potentials have been chosen in such a way that the original action (1) is invariant under global Weyl-scale transformations:

$$\begin{aligned} g_{\mu\nu} &\rightarrow \lambda g_{\mu\nu}, \quad \Gamma_{\nu\lambda}^\mu \rightarrow \Gamma_{\nu\lambda}^\mu, \quad \varphi \rightarrow \varphi + \frac{1}{\alpha}\ln\lambda, \quad \sigma \rightarrow \sigma, \\ A_{\mu\nu\kappa} &\rightarrow \lambda A_{\mu\nu\kappa}, \quad B_{\mu\nu\kappa} \rightarrow \lambda^2 B_{\mu\nu\kappa}, \quad H_{\mu\nu\kappa} \rightarrow H_{\mu\nu\kappa} \end{aligned} \quad (6)$$

For the same reason we have multiplied by an appropriate exponential factor the scalar kinetic term in $L^{(2)}$ and also R and R^2 couple to the two different modified measures because of the different scalings of the latter.

Let us note that the requirement about the global Weyl-scale symmetry (6) uniquely fixes the structure of the non-Riemannian-measure gravity-matter action (1) (recall that the gravity terms R and R^2 are taken in the first order (Palatini) formalism).

Let us also note that the global Weyl-scale symmetry transformations defined in (6) are *not* the standard Weyl-scale (or conformal) symmetry known in ordinary conformal field theory. It is straightforward to check that the dimensionful parameters α, f_1, f_2 present in (3)-(4) do *not* spoil at all the symmetry given in (6). In particular, unlike the standard form of the Weyl-scale transformation for the metric the transformation of the scalar field φ is not the canonical scale transformation known in standard conformal field theories. In fact, as shown in the second Ref.[14] in the context of a simpler than (1) model with only one non-Riemannian measure, upon appropriate φ -dependent conformal rescaling of the metric together with a scalar field redefinition $\varphi \rightarrow \phi \sim e^{-\varphi}$, one can transform the latter

model into Zee's induced gravity model [23], where its pertinent scalar field ϕ transforms multiplicatively under the above scale transformations as in standard conformal field theory.

The equations of motion resulting from the action (1) are as follows. Variation of (1) w.r.t. affine connection $\Gamma_{\nu\lambda}^\mu$:

$$\int d^4x \sqrt{-g} g^{\mu\nu} \left(\frac{\Phi_1}{\sqrt{-g}} + 2\epsilon \frac{\Phi_2}{\sqrt{-g}} R \right) (\nabla_\kappa \delta \Gamma_{\mu\nu}^\kappa - \nabla_\mu \delta \Gamma_{\kappa\nu}^\kappa) = 0, \quad (7)$$

shows, following the analogous derivation in the Ref.[14], that $\Gamma_{\nu\lambda}^\mu$ becomes a Levi-Civita connection:

$$\Gamma_{\nu\lambda}^\mu = \Gamma_{\nu\lambda}^\mu(\bar{g}) = \frac{1}{2} \bar{g}^{\mu\kappa} (\partial_\nu \bar{g}_{\lambda\kappa} + \partial_\lambda \bar{g}_{\nu\kappa} - \partial_\kappa \bar{g}_{\nu\lambda}), \quad (8)$$

w.r.t. to the Weyl-rescaled metric $\bar{g}_{\mu\nu}$:

$$\bar{g}_{\mu\nu} = (\chi_1 + 2\epsilon\chi_2 R) g_{\mu\nu}, \quad \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}}, \quad \chi_2 \equiv \frac{\Phi_2(B)}{\sqrt{-g}}. \quad (9)$$

Variation of the action (1) w.r.t. auxiliary tensor gauge fields $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$ and $H_{\mu\nu\lambda}$ yields the equations:

$$\partial_\mu [R + L^{(1)}] = 0, \quad \partial_\mu [L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}}] = 0, \quad \partial_\mu \left(\frac{\Phi_2(B)}{\sqrt{-g}} \right) = 0, \quad (10)$$

whose solutions read:

$$\frac{\Phi_2(B)}{\sqrt{-g}} \equiv \chi_2 = \text{const}, \quad R + L^{(1)} = -M_1 = \text{const}, \quad L^{(2)} + \epsilon R^2 + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = \text{const}. \quad (11)$$

Here M_1 and M_2 are arbitrary dimensionfull integration constants and χ_2 is an arbitrary dimensionless integration constant.

The first integration constant χ_2 in (11) preserves global Weyl-scale invariance (6), whereas the appearance of the second and third integration constants M_1 , M_2 signifies *dynamical spontaneous breakdown* of global Weyl-scale invariance under (6) due to the scale non-invariant solutions (second and third ones) in (11).

To this end let us recall that classical solutions of the whole set of equations of motion (not only those of the scalar field(s)) correspond in the semiclassical limit to ground-state expectation values of the corresponding fields. In the present case some of the pertinent classical solutions (second and third Eqs.(11)) contain arbitrary integration constants M_1 , M_2 whose appearance makes these solutions non-covariant w.r.t. the symmetry transformations (6). Thus, spontaneous symmetry breaking of (6) is not necessarily originating from some

fixed extrema of the scalar potentials. In fact, as we will see in the next Section below, the (static) classical solutions for the scalar field defined through extremizing the effective Einstein-frame scalar potential (Eq.(27) below) belong to the two infinitely large flat regions of the latter (infinitely large “valleys” of “ground states”), therefore, this does not constitute a breakdown of the shift symmetry of the scalar field (6). Thus, it is the appearance of the arbitrary integration constants M_1 , M_2 , which triggers the spontaneous breaking of global Weyl-scale symmetry (6).

Varying (1) w.r.t. $g_{\mu\nu}$ and using relations (11) we have:

$$\chi_1 \left[R_{\mu\nu} + \frac{1}{2} (g_{\mu\nu} L^{(1)} - T_{\mu\nu}^{(1)}) \right] - \frac{1}{2} \chi_2 \left[T_{\mu\nu}^{(2)} + g_{\mu\nu} (\epsilon R^2 + M_2) - 2R R_{\mu\nu} \right] = 0, \quad (12)$$

where χ_1 and χ_2 are defined in (9), and $T_{\mu\nu}^{(1,2)}$ are the energy-momentum tensors of the scalar field Lagrangians with the standard definitions:

$$T_{\mu\nu}^{(1,2)} = g_{\mu\nu} L^{(1,2)} - 2 \frac{\partial}{\partial g^{\mu\nu}} L^{(1,2)}. \quad (13)$$

Taking the trace of Eqs.(12) and using again second relation (11) we solve for the scale factor χ_1 :

$$\chi_1 = 2\chi_2 \frac{T^{(2)}/4 + M_2}{L^{(1)} - T^{(1)}/2 - M_1}, \quad (14)$$

where $T^{(1,2)} = g^{\mu\nu} T_{\mu\nu}^{(1,2)}$.

Using second relation (11) Eqs.(12) can be put in the Einstein-like form:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{2} g_{\mu\nu} (L^{(1)} + M_1) + \frac{1}{2\Omega} (T_{\mu\nu}^{(1)} - g_{\mu\nu} L^{(1)}) + \frac{\chi_2}{2\chi_1 \Omega} \left[T_{\mu\nu}^{(2)} + g_{\mu\nu} (M_2 + \epsilon(L^{(1)} + M_1)^2) \right], \quad (15)$$

where:

$$\Omega = 1 - \frac{\chi_2}{\chi_1} 2\epsilon (L^{(1)} + M_1). \quad (16)$$

Let us note that (9), upon taking into account second relation (11) and (16), can be written as:

$$\bar{g}_{\mu\nu} = \chi_1 \Omega g_{\mu\nu}. \quad (17)$$

Now, we can bring Eqs.(15) into the standard form of Einstein equations for the rescaled metric $\bar{g}_{\mu\nu}$ (17), i.e., the Einstein-frame gravity equations:

$$R_{\mu\nu}(\bar{g}) - \frac{1}{2} \bar{g}_{\mu\nu} R(\bar{g}) = \frac{1}{2} T_{\mu\nu}^{\text{eff}}, \quad (18)$$

with energy-momentum tensor corresponding (according to (13)):

$$T_{\mu\nu}^{\text{eff}} = g_{\mu\nu} L_{\text{eff}} - 2 \frac{\partial}{\partial g^{\mu\nu}} L_{\text{eff}}, \quad (19)$$

to the following effective Einstein-frame scalar field Lagrangian:

$$L_{\text{eff}} = \frac{1}{\chi_1 \Omega} \left\{ L^{(1)} + M_1 + \frac{\chi_2}{\chi_1 \Omega} \left[L^{(2)} + M_2 + \epsilon (L^{(1)} + M_1)^2 \right] \right\}. \quad (20)$$

In order to explicitly write L_{eff} in terms of the Einstein-frame metric $\bar{g}_{\mu\nu}$ (17) we use the short-hand notation for the scalar kinetic terms:

$$X \equiv -\frac{1}{2} \bar{g}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi, Y \equiv -\frac{1}{2} \bar{g}^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma, \quad (21)$$

and represent $L^{(1,2)}$ in the form:

$$L^{(1)} = \chi_1 \Omega X + \chi_1 \Omega Y - \frac{\mu^2 \sigma^2}{2} \exp\{-\alpha\varphi\} - V, \quad L^{(2)} = \chi_1 \Omega b e^{-\alpha\varphi} X + U, \quad (22)$$

with V and U as in (3)-(4).

From Eqs.(14) and (16), taking into account (22), we find:

$$\frac{1}{\chi_1 \Omega} = \frac{(V + \frac{\mu^2 \sigma^2}{2} \exp\{-\alpha\varphi\} - M_1)}{2\chi_2 \left[U + M_2 + \epsilon (V + \frac{\mu^2 \sigma^2}{2} \exp\{-\alpha\varphi\} - M_1)^2 \right]} \left[1 - \chi_2 \left(\frac{b e^{-\alpha\varphi}}{V + \frac{\mu^2 \sigma^2}{2} \exp\{-\alpha\varphi\} - M_1} - 2\epsilon \right) X \right]. \quad (23)$$

At this point we see that keeping the ϵ contributions will lead to mixed curvaton - dilaton kinetic terms, i.e, XY terms, these will lead to oscillations between these two fields. For simplicity, we want to stick to a more standard curvaton scenario, where the curvaton oscillates, while the inflaton (in our case the dilaton) does not participate in those oscillations. So we consider in what follows $\epsilon = 0$, in a future study the more complex case where $\epsilon \neq 0$ could be studied. Upon substituting expression (23) into (20) we arrive at the explicit form for the Einstein-frame scalar Lagrangian:

$$L_{\text{eff}} = A(\varphi, \sigma) X + B(\varphi) X^2 + Y - U_{\text{eff}}(\varphi, \sigma);, \quad (24)$$

where:

$$\begin{aligned} A(\varphi, \sigma) &\equiv 1 + \left[\frac{1}{2} b e^{-\alpha\varphi} \right] \frac{V + \frac{\mu^2 \sigma^2}{2} \exp\{-\alpha\varphi\} - M_1}{U + M_2} \\ &= 1 + \left[\frac{1}{2} b e^{-\alpha\varphi} \right] \frac{f_1 e^{-\alpha\varphi} + \frac{\mu^2 \sigma^2}{2} \exp\{-\alpha\varphi\} - M_1}{f_2 e^{-2\alpha\varphi} + M_2}, \end{aligned} \quad (25)$$

and

$$\begin{aligned} B(\varphi) &\equiv \chi_2 \frac{-\frac{1}{4}b^2 e^{-2\alpha\varphi}}{U + M_2} \\ &= \chi_2 \frac{-\frac{1}{4}b^2 e^{-2\alpha\varphi}}{f_2 e^{-2\alpha\varphi} + M_2}, \end{aligned} \quad (26)$$

whereas the effective scalar field potential reads:

$$U_{\text{eff}}(\varphi, \sigma) \equiv \frac{(V + \frac{\mu^2 \sigma^2}{2} \exp\{-\alpha\varphi\} - M_1)^2}{4\chi_2 [U + M_2]} = \frac{\left(f_1 e^{-\alpha\varphi} + \frac{\mu^2 \sigma^2}{2} \exp\{-\alpha\varphi\} - M_1\right)^2}{4\chi_2 [f_2 e^{-2\alpha\varphi} + M_2]}, \quad (27)$$

where in the last step the explicit form of V and U (3)-(4) are inserted.

Let us recall that the dimensionless integration constant χ_2 is the ratio of the original second non-Riemannian integration measure to the standard Riemannian one (9).

To conclude this Section let us note that choosing the “wrong” sign of the scalar potential $U(\varphi)$ (Eq.(4)) in the initial non-Riemannian-measure gravity-matter action (1) is necessary to end up with the right sign in the effective scalar potential (27) in the physical Einstein-frame effective gravity-matter action (24). On the other hand, the overall sign of the other initial scalar potential $V(\varphi)$ (Eq.(4)) is in fact irrelevant since changing its sign does not affect the positivity of effective scalar potential (27).

Let us also remark that the effective matter Lagrangian (24) is called “Einstein-frame scalar Lagrangian” in the sense that it produces the effective energy-momentum tensor (19) entering the effective Einstein-frame form of the gravity equations of motion (18) in terms of the conformally rescaled metric $\bar{g}_{\mu\nu}$ (17) which have the canonical form of Einstein’s gravitational equations. On the other hand, the pertinent Einstein-frame effective scalar Lagrangian (24) arises in a non-canonical “k-essence” [7] type form.

A general remark concerning the counting of degrees of freedom is also in order. For this purpose it is crucial that generically the “first order formalism” where the connection is a true independent degree of freedom and the “second order formalism” where the connection is assumed a priori to be the standard Christoffel symbol are really generically different theories, except for the case of the Einstein-Hilbert action, so the notion that these are just to different formulations of the same theory is justified only the case we deal with the Einstein-Hilbert action, for generalizations, theories with a similar looking lagrangian are inequivalent in the two formulations, even the counting of degrees of freedom is different. For example introducing non linear curvature terms or modified measures does not change

the number of degrees of freedom in the first order formulation, where the new measure can be solved in terms of the Riemannian times a function of the matter fields, this is not true if we were to consider the second order formalism. In the same way, there is no increase in degrees of freedom by the introduction of non linear curvature terms in the first order formulation, like in our case the introduction of R^2 terms, again, this is not true in the second order formulation of the theory, where the introduction of non linear curvature terms does indeed changes the order of the equations and therefore causes an increase in degrees of freedom of the theory. In particular in our case, using the first order formulation, the choice $\epsilon = 0$ does not represent a change in the number of degrees of freedom of the theory, just merely a particular parameter choice that simplifies the equations.

III. INFLATIONARY PHASE AND DARK ENERGY PHASE FROM FLAT REGIONS OF THE EFFECTIVE SCALAR POTENTIAL

Depending on the sign of the integration constant M_1 we obtain two types of shapes for the effective scalar potential $U_{\text{eff}}(\varphi)$ (27). This sign determines whether the effective potential has a zero or not. The crucial feature of $U_{\text{eff}}(\varphi)$ is the presence of two infinitely large flat regions, or more precisely φ independent regions – for large negative and large positive values of the scalar field φ . For large negative values of φ , which will characterize the inflationary region, we have for the effective potential and the coefficient functions in the Einstein-frame scalar Lagrangian (24)-(27), when keeping up to quadratic terms in σ only :

$$U_{\text{eff}}(\varphi) \simeq \frac{f_1^2/f_2}{4\chi_2} + \frac{m^2\sigma^2}{2} \equiv \frac{f_1^2/f_2}{4\chi_2} + U(\sigma);, \quad (28)$$

$$A(\varphi, \sigma) \simeq A_{(-)} \equiv 1 + \frac{1}{2}bf_1/f_2 + \frac{m^2bf_1\sigma^2}{4f_2}, \quad B(\varphi) \simeq B_{(-)} \equiv -\chi_2b^2/4f_2. \quad (29)$$

In the second flat region for large positive φ , which will characterize the present slowly accelerated phase of the universe, we obtain:

$$U_{\text{eff}}(\varphi) \simeq U_{(+)} \equiv \frac{M_1^2/M_2}{4\chi_2}, \quad (30)$$

$$A(\varphi) \simeq A_{(+)} \equiv 1, \quad B(\varphi) \simeq B_{(+)} \equiv 0, \quad (31)$$

where curvaton mass that appears in (28) in the first flat region in the minus region which is obtained is

$$m^2 = \mu^2 \frac{f_1}{2\chi_2 f_2}. \quad (32)$$

This is the mass for the curvaton relevant to the reheating of the universe. Concerning the magnitude of the Dark Energy Density, if we take the integration constant $\chi_2 \sim 1$, and if we choose the scales of the scale symmetry breaking integration constants $|M_1| \sim M_{EW}^4$ and $M_2 \sim M_{Pl}^4$, where M_{EW} , M_{Pl} are the electroweak and Plank scales, respectively, we are then naturally led to a very small vacuum energy density $U_{(+)} \sim M_1^2/M_2$ of the order:

$$U_{(+)} \sim M_{EW}^8/M_{Pl}^4 \sim 10^{-120} M_{Pl}^4, \quad (33)$$

which is the right order of magnitude for the present epoche's vacuum energy density. In the present paper we will be mostly concerned with the $\varphi \rightarrow -\infty$ region.

It is interesting to think whether there is any strong motivation to choose $M_1 \sim M_{EW}^4$ and $M_2 \sim M_{Pl}^4$ for explaining the smallness of the Dark Energy Density. This kind of effect is generally present in modified measure theories where the vacuum energy appears as some constant square divided by another constant as discussed first in the first two papers in ref. [14]. What is suggestive about this type of equation is that it resembles the famous "see saw mechanism" used to obtain a small mass for the neutrino, not by fine tuning something to be very small, but rather, suppressing the neutrino mass by a big scale that enters as a denominator that enters in the expression of the diagonalized mass eigenvalues [26]. So as in neutrino physics, the see-saw mechanism is widely employed to understand the tiny masses of the known neutrinos. In our case, we want to relate a see saw effect to understand the smallness of the cosmological constant. In the particle physics case, small masses in the see saw mechanism, the crucial issue is the diagonalization of the mass matrix and in the case of our case the relevant process analogous to the diagonalization of a mass matrix is the transition to the Einstein frame, where a see saw formula for the vacuum energy is obtained. Finally a natural choice for the choices for M_1 and M_2 must be determined in terms of the fundamental mass scales which we know are present in nature, which lead us naturally to the choice of $M_1 \sim M_{EW}^4$ and $M_2 \sim M_{Pl}^4$ for explaining the smallness of the Dark Energy Density.

In this $\varphi \rightarrow -\infty$ region we will study the cosmological evolution. To this end let us recall

the standard Friedman-Lemaitre-Robertson-Walker space-time metric :

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (34)$$

and the associated Friedman equations (recall the presently used units $G_{\text{Newton}} = 1/16\pi$):

$$\frac{\ddot{a}}{a} = -\frac{1}{12}(\rho + 3p) \quad , \quad H^2 + \frac{K}{a^2} = \frac{1}{6}\rho \quad , \quad H \equiv \frac{\dot{a}}{a} \quad , \quad (35)$$

describing the universe' evolution. Here:

$$\rho = \frac{1}{2}A(\varphi, \sigma) \dot{\varphi}^2 + \frac{1}{2}\dot{\sigma}^2 + \frac{3}{4}B(\varphi) \dot{\varphi}^4 + U_{\text{eff}}(\varphi, \sigma) \quad , \quad (36)$$

$$p = \frac{1}{2}A(\varphi, \sigma) \dot{\varphi}^2 + \frac{1}{2}\dot{\sigma}^2 + \frac{1}{4}B(\varphi) \dot{\varphi}^4 - U_{\text{eff}}(\varphi, \sigma), \quad (37)$$

are the energy density and pressure produced by the scalar fields $\varphi = \varphi(t)$ and $\sigma = \sigma(t)$.

The effective action for this cosmological "mini superspace" is

$$S = \int dt a^3 p, \quad (38)$$

with p given by (37). In the limit $\varphi \rightarrow -\infty$, A , B and U_{eff} become φ independent although some σ dependence remains, therefore the above action with p given by (37) acquires the symmetry $\varphi \rightarrow \varphi + \text{constant}$, which means that the canonically conjugate momentum associated to φ is a conserved quantity, that is

$$a^3(A\dot{\phi} + B\dot{\phi}^3) = C_k, \quad (39)$$

where the quantity C_k is a constant.

IV. KINETIC EPOCH

During the kinetic regime, the dynamics of the Friedman-Robertson-Walker cosmology for our model becomes, taking the asymptotically constant values of the coefficients A and B , given by (29), neglecting also the σ dependence of A , then we have,

$$\ddot{\phi}[A + 3B\dot{\phi}^2] + 3H\dot{\phi}[A + B\dot{\phi}^2] = 0, \quad (40)$$

and

$$6H^2 = \rho_{\phi_k}, \quad (41)$$

where the kinetic energy density of the scalar field, ρ_{ϕ_k} , is defined as

$$\rho_{\phi_{kin}} = \frac{A}{2} \dot{\phi}^2 + \frac{3B}{4} \dot{\phi}^4. \quad (42)$$

We obtain a first integral of Eq. (40) given by

$$A\dot{\phi} + B\dot{\phi}^3 = \frac{C_k}{a^3}, \quad (43)$$

where C_k is an integration constant and is defined as

$$C_k = a_k^3 [A\dot{\phi}_k + B\dot{\phi}_k^3].$$

We note that Eq.(43) agrees with the Noether conserved quantity defined before. Here, the quantities $\dot{\phi}_k$ and a_k correspond to the values at the beginning of the kinetic epoch for the quantity $\dot{\phi}$ and the scale factor a , respectively.

The real solution of Eq.(43) can be written as

$$\dot{\phi} = \dot{\phi}(a) = \frac{[9\tilde{B}a^{-3} + \sqrt{12\tilde{A}^3 + 81\tilde{B}^2a^{-6}}]^{2/3} - 2^{2/3}3^{1/3}\tilde{A}}{2^{1/3}3^{2/3}[9\tilde{B}a^{-3} + \sqrt{12\tilde{A}^3 + 81\tilde{B}^2a^{-6}}]^{1/3}}, \quad (44)$$

where $\tilde{A} = \frac{A}{B}$, and $\tilde{B} = \frac{C_k}{B}$.

In this form, we can rewritten Eq.(44) as

$$\dot{\phi} = \dot{\phi}_k \mathcal{F}_a = \dot{\phi}_k \mathcal{F}(a/a_k), \quad (45)$$

where the function \mathcal{F}_a is given by

$$\mathcal{F}_a = \mathcal{F}(a/a_k) = \left(\frac{[9\tilde{B}a^{-3} + \sqrt{12\tilde{A}^3 + 81\tilde{B}^2a^{-6}}]^{2/3} - 2^{2/3}3^{1/3}\tilde{A}}{2^{1/3}3^{2/3}[9\tilde{B}a^{-3} + \sqrt{12\tilde{A}^3 + 81\tilde{B}^2a^{-6}}]^{1/3}} \right) \times$$

$$\left(\frac{2^{1/3}3^{2/3}[9\tilde{B}a_k^{-3} + \sqrt{12\tilde{A}^3 + 81\tilde{B}^2a_k^{-6}}]^{1/3}}{[9\tilde{B}a_k^{-3} + \sqrt{12\tilde{A}^3 + 81\tilde{B}^2a_k^{-6}}]^{2/3} - 2^{2/3}3^{1/3}\tilde{A}} \right),$$

such that $\mathcal{F}_a = \mathcal{F}(a/a_k) |_{a=a_k} = 1$.

Now combining Eqs.(42) and (45), we find an explicit relation for the kinetic energy density in terms of the scale factor a

$$\rho_{\phi_{kin}} = \frac{A}{2} \dot{\phi}_k^2 \mathcal{F}_a^2 + \frac{3B}{4} \dot{\phi}_k^4 \mathcal{F}_a^4, \quad (46)$$

and the Hubble parameter during the kinetic epoch can be written as

$$H_{kin} = \sqrt{\frac{\rho_{kin}}{6}} = \sqrt{\frac{1}{6}} \dot{\phi}_k \mathcal{F}_a \left[\frac{A}{2} + \frac{3B}{4} \dot{\phi}_k^2 \mathcal{F}_a^2 \right]^{1/2}. \quad (47)$$

In the following we will analyze the dynamic of the curvaton field, σ , through different scenario. From these scenarios we will find some constraints of the parameters in our model.

Initially, we consider that the energy density ρ , of the inflaton field, is the dominant component when it is contrasted with the curvaton energy density, ρ_σ . In the next scenario, the curvaton field σ oscillates around the minimum of its effective potential $U(\sigma)$. Its energy density developed as a nonrelativistic matter and in the kinetic scenario, the universe stays inflaton-dominated. In the last stage occurs the decay of the curvaton field into radiation, and therefore the big-bang model is obtained.

For the curvaton field σ we assumed that obeys the Klein-Gordon equation, in which the effective potential associates to curvaton field can be written as $U(\sigma) = \frac{m^2 \sigma^2}{2}$, where m corresponds to the curvaton mass, see Eq.(32).

During the inflationary epoch is assumed that the curvaton mass m satisfied the condition $m \ll H_e$, where H_e corresponds to the Hubble factor at the end of inflation. Recalled that the inflationary evolution in our model is described in detail in Ref.[11]. In the inflationary scenario, the curvaton field σ would roll down its potential until its kinetic energy vanished. In this situation the curvaton field assumes a constant value, in which $\sigma_* \approx \sigma_e$. In the following, the subscript $*$ refers to the epoch when the cosmological scales exit the horizon.

Following Ref.[27], we considered that during the kinetic epoch the Hubble factor decreases so that its value is similar to the curvaton mass, then $m \simeq H_{kin}$. In this way, considering Eq.(47), we can written

$$m \simeq \sqrt{\frac{1}{6}} \dot{\phi}_k \mathcal{F}_{a_m} \left[\frac{A}{2} + \frac{3B}{4} \dot{\phi}_k^2 \mathcal{F}_{a_m}^2 \right]^{1/2}, \quad (48)$$

or equivalently

$$\mu^2 \frac{f_1}{\chi_2 f_2} \simeq \frac{1}{3} \dot{\phi}_k^2 \mathcal{F}_{a_m}^2 \left[\frac{A}{2} + \frac{3B}{4} \dot{\phi}_k^2 \mathcal{F}_{a_m}^2 \right],$$

where the ‘m’ label corresponds to the quantities at the time during the kinetic epoch when the curvaton mass is of the order of H_{kin} , and $\mathcal{F}_a|_{a=a_m} = \mathcal{F}_{a_m}$.

For avoiding a period of curvaton-driven inflation, then we considered that $\rho_{\phi_{kin}}|_{a_m} = \rho_{\phi_{kin}}^{(m)} \gg \rho_\sigma(\sim U(\sigma_e) \simeq U(\sigma_*))$. This relation allows us to obtain during the inflationary

scenario a constraint on the values of the curvaton field σ_* i.e., the value of the curvaton field when the cosmological scales exit the horizon. Hence, from Eq.(41), at the moment when $H \simeq m$ we get the restriction

$$\frac{m^2 \sigma_*^2}{2\rho_{\phi_{kin}}^{(m)}} \ll 1, \quad \text{or equivalently} \quad \sigma_*^2 \ll 12. \quad (49)$$

We note that this upper bound for the curvaton field σ_* is similar to that obtained in the standard scalar field [27].

Also, we find that the ratio between the potential energies at the end of inflation becomes

$$\frac{U_e}{V_e} \ll 1, \quad \text{or equivalently} \quad \frac{1}{12} \frac{m^2 \sigma_*^2}{H_e^2} \ll \frac{m^2}{H_e^2}, \quad (50)$$

where we considered that effective potential at the end of inflation is given by $V_e = 6 H_e^2$ together with Eq.(49). In this way, we find that the curvaton mass satisfied the constraint $m \ll H_e$. We note that the condition $m \ll H_e$ is inherent to the nature of the curvaton field, and becomes a fundamental prerequisite for the curvaton mechanism[28].

After the mass of curvaton field becomes $m \simeq H_{kin}$, its energy decays $\rho_\sigma \propto a^{-3}$ (non-relativistic matter), and then we can write

$$\rho_\sigma = \frac{m^2 \sigma_*^2 a_m^3}{2 a^3}. \quad (51)$$

In the following, we will analyze the decay of the curvaton field in two possible different scenarios; the curvaton field decay after domination and the curvaton decay before domination.

V. CURVATON DECAY AFTER DOMINATION

As we have required the curvaton field decay could take place in two different possible scenarios. During the first scenario, the curvaton field decay after domination, and then the curvaton field comes to dominates the cosmic expansion, in which $\rho_\sigma > \rho_\phi$, there must be an instant when the inflaton and curvaton energy densities becomes equivalent. Considering that both densities becomes equivalent and this occurs when $a = a_{eq}$, then from Eqs. (46) and (51) we find

$$\left. \frac{\rho_\sigma}{\rho_{\phi_{kin}}} \right|_{a=a_{eq}} = \frac{m^2 \sigma_*^2 a_m^3}{2 a_{eq}^3} \frac{1}{\dot{\phi}_k^2 \mathcal{F}_{a_{eq}}^2 \left[\frac{A}{2} + \frac{3B}{4} \dot{\phi}_k^2 \mathcal{F}_{a_{eq}}^2 \right]} = 1, \quad (52)$$

where the function $\mathcal{F}_{a_{eq}}$ corresponds to $\mathcal{F}_a|_{a=a_{eq}} = \mathcal{F}_{a_{eq}}$.

From Eqs.(51) and (52), we find a relation for the Hubble factor, $H(a = a_{eq}) = H_{eq}$, as a functions of the curvaton parameters, together with the ratio of the scale factor at different times

$$H_{eq} = \sqrt{\frac{\sigma_*^2}{12}} \left[\frac{a_m}{a_{eq}} \right]^{3/2} m. \quad (53)$$

Here, we note that this equation coincides with the one found in standard case, see Refs.[27, 31].

Also, we require that the curvaton field decays before of nucleosynthesis, which means $H_{nucl} \sim 10^{-40} < \Gamma_\sigma$, where the decay parameter Γ_σ is constrained by nucleosynthesis. However, we also postulate that the curvaton decay occurs after $\rho_\sigma > \rho_\phi$, together with the condition $\Gamma_\sigma < H_{eq}$. In this form, considering Eq.(53) we can written

$$10^{-40} < \Gamma_\sigma < \sqrt{\frac{1}{12}} \left[\frac{a_m}{a_{eq}} \right]^{3/2} m \sigma_*. \quad (54)$$

Now we will find a constraint of the parameters of our model, by considering the scalar perturbation associated to the curvaton field σ . In this context, the fluctuations of the curvaton field satisfies an analogous differential equation to the inflaton fluctuations, and then we consider that the fluctuations of the curvaton field takes the amplitude $\delta\sigma_* \simeq H_*/2\pi$. From the spectrum of the Bardeen parameter $P_\zeta \simeq H_*^2/(9\pi^2\sigma_*^2) \sim 10^{-9}$ [28], we get

$$P_\zeta \simeq \frac{1}{54\pi^2} \frac{V_*}{\sigma_*^2} = \frac{(f_1 e^{-\alpha\phi_*} - M_1)^2}{216\pi^2 \chi_2 [f_2 e^{-2\alpha\phi_*} + M_2]} \sigma_*^2, \quad (55)$$

in which ϕ_* corresponds to the value of the curvaton field when the cosmological scales exit the horizon. From Ref.[11] the value of ϕ_* is given by

$$e^{-\alpha\phi_*} = \frac{2\alpha M_1}{f_1(1 + bf_1/2f_2)} [C_2 + 2\alpha N], \quad \text{where} \quad C_2 = \sqrt{(1 + bf_1/2f_2)}.$$

Here, we have considered that $e^{-\alpha\phi_e} = \frac{2\alpha M_1}{f_1[1+bf_1/f_2]^{1/2}}$, see Ref.[11], and the quantity N corresponds to the number of e-folds.

Considering the constraint given by Eq.(54) together with the condition in which $a_m < a_{eq}$, we get

$$\Gamma_\sigma < \sqrt{\frac{1}{12}} \left(\frac{m(f_1 e^{-\alpha\phi_*} - M_1)^2}{216\pi^2 \chi_2 P_\zeta [f_2 e^{-2\alpha\phi_*} + M_2]} \right), \quad (56)$$

and we note that this expression gives an upper limit on the parameter Γ_σ when the curvaton decays after domination.

On the other hand, we admit that the reheating occurs before the big-bang nucleosynthesis (BBN) temperature T_{BBN} , in which the temperature of reheating $T_{rh} > T_{BBN}$. However, when the curvaton field decays at the time before the electroweak scale (ew) where the temperature is T_{ew} , then we require that $T_{rh} \sim \Gamma_\sigma^{1/2} > T_{ew} > T_{BBN}$, in which $T_{ew} \sim 10^{-17}$ and $T_{BBN} \sim 10^{-22}$. In this form, considering Eq.(56) we can write

$$\frac{m^{1/2}(f_1 e^{-\alpha\phi_*} - M_1)}{\chi_2^{1/2} [f_2 e^{-2\alpha\phi_*} + M_2]^{1/2}} > (12)^{1/4} (216 P_\zeta)^{1/2} \pi T_{ew} \sim 10^{-20}, \quad (57)$$

and using Eq.(32), the ratio $\mu^{2/3}/\chi_2$ results

$$\frac{\mu^{2/3}}{\chi_2} > 10^{-7}. \quad (58)$$

Here, following Ref.[11] we have considered the values $M_1 = 4 \times 10^{-60}$ (in units of M_{Pl}^4), $M_2 = 4$ (in units of M_{Pl}^4), $b = -0.52$, $\alpha = 1$, $f_1 = 2 \times 10^{-8}$, $f_2 = 10^{-8}$ and the number of e-folds $N = 60$. In particular, considering the range for χ_2 found in Ref.[11] in which $58 \times 10^{-6} < \chi_2 < 74 \times 10^{-3}$, we obtain that the lower bound of the parameter μ becomes $\mu > 6 \times 10^{-13}$. Here, we note that from Eqs.(30) and (66), the effective potential $U_+ = \frac{M_1^2/M_2}{4\chi_2} \sim \frac{10^{-120}}{4\chi_2} > \frac{10^{-198}}{\mu^2}$, and considering that the present day value of the density parameter of dark energy is given by $\Omega_{DE}^0 = \frac{U_+}{H_0^2} \simeq 0.7$ where H_0 denotes of the present Hubble parameter $H_0 \simeq 10^{-61}$ (in units of M_{Pl}), we get $\mu > 10^{-35}$. In this form, we find a lower bound even smaller for the parameter μ , from present dark energy density in relation to the lower bound for μ , obtained from the range of χ_2 in Ref.[11].

VI. CURVATON DECAY BEFORE DOMINATION

In the second scenario the curvaton field decays before it dominates the cosmological expansion. In this form, we require that the curvaton σ decays before that its energy density ρ_σ becomes greater than the energy density of the scalar field ρ_ϕ . Also, during this scenario the mass of the curvaton m is similar to the Hubble parameter, i.e., $m \sim H$, and if the curvaton decays at a moment whenever the decay parameter $\Gamma_\sigma = H(a_d) = H_d$, in which d label corresponds to quantities at the moment when the curvaton field decays. From this

condition and considering Eq.(47), we get

$$\Gamma_\sigma = H_d = \sqrt{\frac{1}{6}} \dot{\phi}_k \mathcal{F}_{a_d} \left[\frac{A}{2} + \frac{3B}{4} \dot{\phi}_k^2 \mathcal{F}_{a_d}^2 \right]^{1/2}. \quad (59)$$

Before that the curvaton field dominates the cosmological expansion, i.e., $\rho_\sigma < \rho_\phi$, the decay parameter Γ_σ satisfies $\Gamma_\sigma > H_{eq}$, and on the other hand, the curvaton mass m becomes important during this scenario, in which $m > \Gamma_\sigma$. In this way, considering Eq.(53) we find

$$\sqrt{\frac{\sigma_*^2}{12}} \left[\frac{a_m}{a_{eq}} \right]^{3/2} < \frac{\Gamma_\sigma}{m} < 1, \quad (60)$$

and these inequalities for the ratio Γ_σ/m , are similar to that reported in Ref.[27].

On the other hand, the spectrum of the Bardeen parameter during this scenario, is given by [33]

$$P_\zeta \simeq \frac{r_d^2}{16\pi^2} \frac{H_*^2}{\sigma_*^2}, \quad (61)$$

where the parameter r_d corresponds to the ratio between the curvaton and inflaton energy densities, evaluated at the moment in which the curvaton decay takes place i.e., $a = a_d$. From Eq.(51) and considering that $H_d = \Gamma_\sigma$, the parameter r_d is given by

$$r_d = \left[\frac{\rho_\sigma}{\rho_\phi} \right]_{a=a_d} = \frac{m^2 \sigma_*^2}{12} \left(\frac{a_k}{a_d} \right)^3 \frac{1}{\Gamma_\sigma^2}. \quad (62)$$

Here we remark that the parameter r_d is associated to two other quantities; the parameter the non-Gaussianity f_{NL} in which $f_{NL} \sim r_d^{-1}$, see Ref.[34], and the ratio the isocurvature and adiabatic amplitudes[39].

One may ask, is there any smoking gun for this scenario? The strongest constraints on the non linear parameter $f_{NL} \gg 1$ will get from the measurements of the CMB sky in future detection, and in this form the non linear parameter would be a smoking gun for the curvaton-two measures theory. In particular if the non Gaussianity is of the local type, the non linear parameter f_{NL}^{local} from the curvaton field considering the Planck data 2015 is given by $f_{NL}^{local} = 2.5 \pm 5.7$ at 68% CL [35], in the case in which there is no import decay of the inflaton field into curvaton particles. Otherwise, if the inflaton field into curvaton particles the non linear parameter f_{NL}^{local} was obtained in Ref.[36], and from Planck results $f_{NL}^{local} \sim O(1)$. However, due to the measurement errors no conclusive affirmation can be made, since the data are preliminaries[35], and therefore the observational ranges on the magnitude of the non linear parameter will progress considerably in the near future.

On the other hand, another conceivable signature in our model is the isocurvature perturbation from the baryon or neutrinos density that is associated to the curvature perturbation, from the curvaton decay. This isocurvature perturbation was studied in Refs.[32][37], and this mechanism of correlation should be measurement, and could become an observable magnitude from CMB observations[38].

Now combining Eqs.(61) and (62) we obtain that the curvaton field σ_* can be written as

$$\sigma_*^2 = \frac{2304\pi^2 P_\zeta}{m^4 H_*^2} \left[\frac{a_d}{a_k} \right]^6 \Gamma_\sigma^4, \quad (63)$$

and by using Eqs.(60) and (63), we find an upper bound for the parameter Γ_σ given by

$$\Gamma_\sigma^3 < \frac{m^3 H_*^2}{1152\pi^2 P_\zeta} \left[\frac{a_k}{a_d} \right]^6. \quad (64)$$

Now considering that the reheating temperature T_{rh} satisfies the constraint $T_{rh} > T_{BBN} \sim 10^{-22}$, with $\Gamma_\sigma > T_{BBN}^2$ we find

$$m H_*^{2/3} \left[\frac{a_k}{a_d} \right]^3 > (1152\pi^2)^{1/3} P_\zeta^{1/3} T_{BBN}^2 \sim 10^{-44}. \quad (65)$$

Considering that $a_d > a_k$, and that the curvature perturbations generated during inflation are due to quantum fluctuations of the curvaton field, where the energy scale is approximately $V_*^{1/4} \approx 10^{15-16}$ GeV (upper bound)[40], we get

$$\frac{\mu}{\chi_2^{1/2}} > (6912\pi^2)^{1/3} \frac{P_\zeta^{1/3} T_{BBN}^2}{V_*^{1/3}} \sqrt{\frac{2f_2}{f_1}} \sim 10^{-39}. \quad (66)$$

Here we have used Eq.(32), and the values $f_1 = 2 \times 10^{-8}$ and $f_2 = 10^{-8}$ from Ref.[11]. This expression gives a lower bound on the parameters $\mu/\chi_2^{1/2}$, during the curvaton field decay before domination. In particular, using the range for χ_2 from Ref.[11] in which $58 \times 10^{-6} < \chi_2 < 74 \times 10^{-3}$, we find that the lower bound of the parameter μ is given by $\mu > 2 \times 10^{-40}$.

VII. CONCLUSIONS

In this paper we have studied in detail the curvaton reheating mechanism in a scale invariant two measures theory. In this framework the responsible for the reheating of the universe as well as the spectrum of curvature perturbations is the curvaton field σ .

We have considered that our model involves two scalar matter fields, a dilaton, that transforms under scale transformations and it drives the expansion of the universe, and another scalar, which does not transform under scale transformations and it play the role of a curvaton field, with an effective mass during the reheating of the universe.

Considering the curvaton reheating mechanism we have examined two possible scenarios. Firstly, the curvaton field dominates the universe after it decays, and in the second scenario the curvaton field decays before domination. In these scenarios, we have obtained constraints for the values of the decay parameter Γ_σ which are represented by Eqs.(56) and (64), respectively.

From the stage in which the curvaton field decays after its dominates, we have obtained a lower limit for the ratio $\mu^{2/3}/\chi_2 > 10^{-7}$. Here, we have considered Eq. (57), together with the values found in Ref.[11] , in which $M_1 = 4 \times 10^{-60}$, $M_2 = 4$, $b = -0.52$, $\alpha = 1$, $f_1 = 2 \times 10^{-8}$, $f_2 = 10^{-8}$ and the number of e-folds $N = 60$. In particular, considering the range for χ_2 found in Ref.[11] in which $58 \times 10^{-6} < \chi_2 < 74 \times 10^{-3}$, we have obtained that the lower bound of the parameter μ becomes $\mu > 6 \times 10^{-13}$.

In the second scenario, we could estimate the lower bound for the ratio $\mu/\chi_2^{1/2} > 10^{-39}$. Here we have considered Eq.(32), and the values $f_1 = 2 \times 10^{-8}$ and $f_2 = 10^{-8}$ from Ref.[11]. Also, in particular, using the range for χ_2 from Ref.[11] we have found that the lower bound of the parameter μ is given by $\mu > 2 \times 10^{-40}$.

One may ask, is there any smoking gun for this scenario? The strongest constraints on the non linear parameter $f_{NL} \gg 1$ will get from the measurements of the CMB sky in future detection, and in this form the non linear parameter would be a smoking gun for the curvaton-two measures theory. In particular if the non Gaussianity is of the local type, the non linear parameter f_{NL}^{local} from the curvaton field considering the Planck data 2015 is given by $f_{NL}^{local} = 2.5 \pm 5.7$ at 68% CL [35], in the case in which there is no import decay of the inflaton field into curvaton particles. Otherwise, if the inflaton field into curvaton particles the non linear parameter f_{NL}^{local} was obtained in Ref.[36], and from Planck results $f_{NL}^{local} \sim O(1)$. However, due to the measurement errors no conclusive affirmation can be made, since the data are preliminaries[35], and therefore the observational ranges on the magnitude of the non linear parameter will progress considerably in the near future.

In both scenarios the values for the couplings considered allow an emergent non singular scenario followed by inflation as studied in Ref.[11], since the solutions for the emergent

phase found in Ref.[11] hold as solutions of the model enriched by the curvaton that we have added now in the case $\epsilon = 0$ and neglecting the curvaton (i.e. considering $\sigma = 0$ or negligible in the early emergent phase).

Finally we discuss generalizations of the model, through the effect of higher curvature terms, where inflaton and curvaton can have coupled oscillations. Indeed, as we have mentioned, for the case of $\epsilon = 0$, there are no mixed curvaton -inflaton kinetic terms, but for any non vanishing value of ϵ , those terms will appear, and they will induce inflaton - curvaton oscillations, a subject of interest for future studies.

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